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Editorial Board

THE PROBLEM OF DEBT STABILIZATION: AN ALTERNATIVE APPROACH

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Abstract. The paper deals with the problems of fiscal sustainability and the stabilization of the public debt. It criticizes the mainstream approach to these issues and argues that an approach inspired by Domar's contribution (1944) is preferable. While mainstream analyses of debt stabilization are based on the hypothesis that the economy's growth rate is independent of public spending and its composition, in the paper this hypothesis is removed. The growth rate is made dependent on the composition of public spending. In this analytical context, ensuring the stabilization of the ratio of the public debt to the GDP does not necessarily requires running a primary surplus, which instead is the fundamental mainstream conclusion when the interest rate on the debt is higher than the economy's growth rate. The debt ratio can be stabilized through increases in the economy's growth rate caused by adequate changes in the composition of public expenditure.
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1. Introduction

The sustainability of fiscal policy, and the related problem of the stabilization of the ratio of the public debt to the GDP, are old questions. In the 1940s, Domar (1944) provided a pioneering contribution to the analysis of these issues. More recently, in the 1990s, Blanchard et al. (1990) have offered what they call 'new answers' to the problem of fiscal sustainability. The approach followed by Blanchard and his co-authors represents the current standard approach in the mainstream literature on the topic.[1]

The old question of fiscal sustainability continues to be a major concern for the economics profession, especially in the context of the present world economic situation. As a growing number of economists call for significant fiscal interventions to help the economies to overcome, or avoid, the dangers of a deep recession (see, for example Spilimbergo et al. (2008)), they at the same time stress that such interventions should be such that fiscal sustainability is guaranteed in the medium term. A fiscal policy is sustainable if it does not make the ratio of the public debt to the GDP rise indefinitely. Although, in the short term, the stabilization of the economy can imply an increase of the ratio of the

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[1] On the current mainstream approach to fiscal policy and fiscal sustainability, see also Blanchard and Fisher (1993, pp.52-8 and 126-35) and Minford and Peel (2002, pp. 182-211).

public debt to the GDP, in the longer term the adopted fiscal policy must be such that the debt ratio is stabilized or reduced. Both a high debt ratio and its increasing over time are regarded as damaging to the economy. In this framework, the most virtuous policy for the stabilization of the debt ratio is one that gives rise to surpluses of the government's primary budget.

The present paper does not intend to enter into a thorough discussion and criticism of the current mainstream approach to the public debt, fiscal policy and debt stabilization. More limitedly, the paper concentrates on the stabilization of the public debt ratio to show that it must not necessarily be implemented through running primary surpluses. The stabilization can be obtained through adequate changes in the composition of the public expenditure while the public primary budget is left unchanged. The problem of debt stabilization, although of a more limited nature than the problem of the public debt and fiscal policy in general, is however of great relevance especially in the context of the present crisis, which is very likely to bring about growing public deficits and growing debt ratios in most economies. The way in which the debt ratios will be stabilized, or reduced, in the future is then important.

The current mainstream approach develops the analysis of fiscal sustainability and debt stabilization under the hypothesis that the growth rate is independent of public spending. Instead, the analysis carried out in this paper is based on the hypothesis that the economy's growth rate is dependent on public spending, namely its composition. This approach is inspired by Domar's original contributions to the problem of the public debt and to the theory of growth. Domar argued that the problem of the public debt ratio to the GDP should be tackled by trying to make the denominator grow more rapidly rather than by merely trying to reduce the numerator. A higher growth rate of the GDP can be obtained also through public spending. More precisely, the economy's growth rate is an increasing function of 'productive' public expenditures, i.e. those that contribute to increase the overall productive capacity and/or the efficiency of production.[2]

The analysis of debt stabilization, carried out under the hypothesis of a positive functional relation between public spending and the growth rate, yields results that are different from those usually obtained by more traditional approaches. The most important conclusion of the present analysis is that the stabilization of the ratio of the public debt to GDP does not necessarily require running a primary surplus, which instead is the fundamental mainstream conclusion when the interest rate on the debt is higher than the economy's growth rate. In the framework proposed here, an adequate reduction of the share of the fiscal revenue devoted to current spending associated with an increase in public investment make the economy's growth rate increase and become higher than the interest rate. In this situation, the debt ratio can be stable without the necessity to run a primary surplus.

The paper is organized as follows. Section 2 briefly illustrates the basic elements of the mainstream approach to fiscal sustainability and debt stabilization; section 3 offers a short presentation of Domar's original contribution to the problem of the public debt. Section 4 proceeds to develop a generalized Domar growth model with a public sector.

[2]Although within a different analytical framework, Goldsmith (2008) has recently obtained similar results concerning the relationship between the composition of public spending and the economy's growth rate. See also Sardoni and Palazzi (2000).

In section 5, this generalized growth model is used to deal with the problem of debt stabilization. Finally, section 6 concludes with some policy considerations as well as some analytical and methodological qualifications.

2. Fiscal sustainability and debt stabilization: the mainstream approach

In a world in which the so-called Ricardian equivalence holds, there is no concern for the amount of public debt. The size of the public debt does not affect the economy as the reduction in the public saving is compensated for by the increase in the private saving, so that total saving and investment are left unaffected. In mainstream macroeconomics, the concern for the public debt and the problem of fiscal sustainability derives from the rejection of the Ricardian equivalence and from the conviction that the size of the ratio of the public debt to the GDP matters as it affects interest rates, risk premia as well as taxes and, hence, the economy's growth. In this analytical framework, the problem of debt stabilization is relevant.

Fiscal policy is defined as sustainable in so far as it does not lead to increasing ratios of the public debt to the GDP or, conversely, to an increasing tax burden. Central to the discussion of sustainability is the dynamic government budget constraint that, in its general form, is

$$\frac{dB}{dh} = G - T + iB - \frac{dM}{dh} \tag{1}$$

where B is the public nominal debt, G is public expenditure, T is taxes (net of transfers), i is the nominal interest rate, M is the supply of high-powered money, h is time.[3] If, the public debt is partly, or totally, financed by the issuing of high-powered money, i.e. the debt is partly or totally monetized.

The possibility to monetize the public debt, and the related problem of seignorage, although considered in the literature is given much less attention than to the financing of the public debt through borrowing from the private sector of the economy. One of the reasons why the attention is focused on borrowing is that, in most economies, the financing of the debt through the creation of money currently represents a small proportion of the total debt financing. Formally, concentrating on the financing through borrowing means that it is assumed that $\frac{dM}{dh} = 0$.

Once the monetization of the debt is assumed away, (1) can be written in terms of ratios to the GDP (Y) and by introducing the real interest rate:

$$\dot{b} = \frac{db}{dh} = (\gamma - \tau) + (r - g)\, b \tag{2}$$

where $b = \frac{B}{Y}, \gamma = \frac{G}{Y}, \tau = \frac{T}{Y}$, r is the real interest rate on the debt and g is the economy's growth rate.

If it is assumed that the interest rate r is larger than the growth rate g, the ratio of public debt to GDP does not vary from one period to the next ($\dot{b} = 0$) only if $(\tau - \gamma) > 0$, i.e. if the government runs a primary surplus. From (2), it must be

$$(r - g)b = \tau - \gamma \tag{3}$$

[3]Throughout the paper, the analysis is carried out by considering a closed economy, so that the whole stock of the public debt is owned by nationals.

Starting from a situation in which it is $\dot{b} > 0$, the ratio of public debt to GDP can be stabilized through fiscal policies that give rise to a primary surplus,[4] i.e. either by increasing the tax ratio and/or by reducing the expenditure ratio.

The conclusions above derive from analyzing the dynamic government budget constraint under the assumption that the economy's growth rate is independent of the fiscal budget and, more in particular, of public spending. The problem of debt stabilization can be analyzed by removing this assumption and by establishing a functional relation between the public expenditure and the growth rate. This latter analytical approach is inspired by Domar, whose contribution to the analysis of the public debt is outlined in the following section.

3. Domar's contribution to the problem of debt sustainability

The main objective of Domar's pioneering article on the 'burden' of the public debt (1944) was to contrast the opinion that deficit spending leads to an ever-growing public debt, the servicing of which inevitably leads to an increasing tax burden on the economy.[5] For Domar, while all previous analyses underlined the obvious fact that continuous borrowing results in an ever-increasing debt, many tended to overlook that deficit spending affects income.[6]

In order to study the analytical relations between deficit spending, debt and income, Domar considers four different cases.[7] In the first case, it is assumed that income remains constant while, in each period, the government borrows a percentage α of income. It is clear that in this case the ratio of debt to income will grow indefinitely. But, for Domar, 'there is something inherently odd about an economy with a continuous stream of investment expenditures and a stationary national income' (1944, p. 804). Such a case could occur because investment does not have any positive effect on productivity and the number of working hours remains unchanged.[8] The second case is analogous to the first. Now income grows over time but at a constant *absolute* rate ($Y_h = d + fh$). Since the government keeps on borrowing α% of the income, also in this case the ratio of debt to income will grow with no limit (Domar, 1944, p. 806).[9]

The third case is the most important. Domar now assumes that income grows at a 'constant percentage rate' g ($Y_h = de^{gh}$). In this case the growth rate of debt will approach the growth rate of income and, therefore, the ratio of debt to income will tend to a constant value (Domar, 1944, p. 809). More precisely, it will approach the value $\frac{\alpha}{g}$.

It follows that the larger is the rate of growth of income, the lower is the ultimate ratio of debt to income.[10] Thus, the problem of the debt ratio lies in the ability to make income

[4] Or fiscal policies that generate a larger primary surplus if it initially is $0 < (\tau - \gamma) < (r - g)b$.

[5] On Domar's approach to the problem of the public debt, see also Pasinetti (1997).

[6] '... that deficit financing may have some effect on income (...) has received a different treatment. Opponents of deficit financing often disregard it completely, or imply, without any proof, that income will not rise as fast as the debt.' (1944, p. 801).

[7] In all four cases, the interest rate on the debt and the price level are taken as given and constant.

[8] Alternatively, productivity grows but the number of working hours diminishes.

[9] The reasons why such a case could occur are the same as in the first case.

[10] See the mathematical appendix to Domar's article (1944, pp. 823-5). The fourth case is a 'war model', in which the percentage of income borrowed differs between peace times and war times. For brevity, this case is not considered here.

grow rather than in attempting to reduce it without taking account of the effects of such a reduction on income[11] A certain growth rate of income can be achieved if aggregate demand grows at that rate and, at the same time, a sufficient amount of the expenditures is directed toward 'increasing the efficiency of production, so as to allow the required volume of monetary expenditures to take place without a rise in prices.' (Domar, 1944, p. 820). The government can contribute to increasing the economy's growth rate by converting part of the private income that it absorbs through taxation into productive expenditures. For simplicity, such expenditures can be thought of as public investment, as opposed to current expenditures seen as unproductive.[12] The analysis carried out in sections 4 and 5 below is in the spirit of Domar's approach. However, whereas Domar deals with the relation between the growth rate and public 'productive' spending only at an intuitive level, the model of the present paper is based on an explicit functional relationship between the growth rate and the composition of public spending.

4. A generalized Domar growth model

As we saw in the previous section, Domar argued that public spending can positively affect the economy's growth rate and, hence, contribute to lowering the ratio of the public debt to GDP. Domar, however, did not provide analytical support for his intuition. In this section, we develop Domar's intuition by introducing a functional relationship between public spending and the growth rate. In order to do so, we start from Domar's original model of growth (Domar, 1946).

In the original Domar's model, the economy's equilibrium growth rate is

$$g = s\sigma$$

s is the private propensity to save and σ is the potential social average investment productivity. It is

$$\sigma = \frac{\frac{dP}{dh}}{I} = \frac{P'}{I}$$

so that $P' = \sigma I$ ($\frac{dP}{dh} = P'$ is the increase in aggregate potential capacity associated with investment I).

Let us now introduce the public sector into the model. Let the average tax rate be denoted by t, so that the government's total revenue is tY. Total public expenditure is distinguished between current spending (C_g) and capital spending (I_g). If $(C_g+I_g) > tY$, the government experiences a primary deficit and borrows from the private sector.

[11]'Now, some economic and political circles are burning with a desire to reduce the debt burden (...). They recognize no other method of achieving their goal but by reducing the absolute size of the debt; that the government must stop borrowing is of course taken for granted. They should beware, however, lest the policies they advocate exert such a depressing effect on the national income as to result in an actually heavier debt burden, even though they succeed in paying off a part of the debt.' (Domar, 1944, pp. 815-6).

[12]But Domar is careful to point out that the distinction between investment and current expenditure may be misleading: 'As a matter of fact, the term "investment expenditures" may be misleading, because it is too closely associated with steel and concrete. If healthier people are more productive, expenditures on public health satisfy these requirements. The same holds true for expenditures on education, research, flood control, resource development and so on.' (1944, p. 820).

The analysis of the relationship between public spending and the growth rate can be carried out in different ways; our analysis is developed by introducing the following assumptions and hypotheses. It is assumed that all public capital expenditures contribute to the increase of the aggregate potential capacity, while current expenditures do not. It is also hypothesized that public current spending is a share $0 < a < 1$ of total revenue, while public capital expenditure is taken as independent of total revenue.

The economy depicted above is in equilibrium when

$$Y' = \frac{dY}{dh} = \frac{dP}{dh} = P'$$

That is to say, the economy is in equilibrium when the increase in the aggregate demand $Y' = (1-s)(1-t)Y' + I'_p + atY' + I'_g$ is equal to the increase in the productive capacity (P') generated by private investment (I_p) and public investment (I_g).

We then obtain the equilibrium growth rate,

$$g = [s(1-t) + t(1-a)]\sigma \tag{4}$$

g, of course, is also the equilibrium growth rate of investment that, in our case, is the sum of public investment and private investment.

In the original Domar model, the higher is the propensity to save, the higher is the equilibrium growth rate. The rationale of this result is simple: the higher is the propensity to save, the larger must be the investment that ensures the equality of aggregate demand to aggregate supply; but the larger is investment, the larger is its growth rate and the larger is the economy's growth rate. The same basic rationale applies to an economy with a public sector. In this case, given σ, the equilibrium growth rate is increasing in $s(1-t) + t(1-a)$, which is the economy's overall marginal propensity to save. The higher is this propensity, the higher must be the equilibrium growth rate of investment and, hence, the higher is the economy's equilibrium growth rate.[13]

What is interesting is that $[s(1-t) + t(1-a)]$ is increasing in t if it is

$$a < (1-s) \tag{5}$$

a can be interpreted as the public propensity to consume, so that condition (5) tells us that an increase in the tax rate is associated to a higher equilibrium growth rate of the economy if the public propensity to consume is lower than the private propensity to consume. In fact, if (5) holds, an increase in the tax rate t necessarily implies a higher overall propensity to save and the equilibrium requires a higher growth rate of investment and, hence, a higher growth rate of the economy.

The overall propensity to save is also inversely related to a. Therefore, from algebra, given t, the economy's growth rate is maximum at $a = 0$, i.e. when all public expenditure is devoted to capital formation. This, however, does not mean that the achievement of such a maximum growth rate is economically and/or socially feasible or desirable. A certain amount of current spending is obviously necessary for running the public sector itself, while other current expenditures are devoted to fundamental state functions like

[13]The equilibrium relation between the propensity to save, investment and growth rate does not imply any causality relation. In other words, it does not mean that saving determines investment. In terms of causality, it is the level of investment that determines the level of saving or, which is the same, the growth rate of investment determines the growth rate of saving.

defense, justice, etc.[14] Therefore, here it is assumed that the ratio of current spending to total revenue is always larger than zero.

In conclusion, any change in t and/or a that makes the overall propensity to save higher requires a higher equilibrium growth rate of investment and of the economy. In the analysis of debt stabilization carried out in the next section, all changes of the overall propensity to save are assumed to be generated by changes in a, the share of total revenue devoted to current spending; the tax rate t is left unvaried throughout.

5. Debt stabilization in the generalized growth model

5.1. **The average productivity of investment is constant.** The present model assumes that public investment contributes to the growth of productive capacity (P). For now, we also assume that the ratio of the increase in P to investment is the same in the public and in the private sector. Therefore

$$P' = \sigma(I_g + I_p)$$

Once the hypothesis that the economy's growth rate depends on the share of public revenue devoted to current public expenditure is introduced, the conclusion that the debt ratio can be stabilized only by running a primary surplus does not necessarily follow any longer. The fact that the growth rate depends on the value taken by a suggests that it could be possible to stabilize the ratio of the public debt to the GDP through a positive variation of g.

Since, from the model of section 4, it is[15]

$$Y = \frac{I_g + I_p}{s(1-t) + t(1-a)} = \frac{I_g + I_p}{\Gamma - at}$$
$$\Gamma = s(1-t) + t > 0$$

and

$$g = [s(1-t) + t(1-a)]\sigma = (\Gamma - at)\sigma)$$

(3) can be written as

$$[r - (\Gamma - at)\sigma] \frac{B(\Gamma - at)}{I_g + I_p} = t - \left[at + \frac{I_g(\Gamma - at)}{I_g + I_p}\right] \tag{6}$$

Let us consider a case in which the government's objective is to stabilize the ratio of the public debt to GDP by leaving the primary budget unchanged. Suppose that, initially, it is $\dot{b} = D > 0$, i.e. the debt ratio is increasing. The problem to solve is the

[14]Moreover, the flow of capital expenditures is generally associated with a certain flow of current expenditures; for example, those current expenditures that are necessary for the maintenance of the infrastructures realized through investment expenditures (Sardoni and Palazzi, 2000, pp. 158-63). This type of current expenditures, however, can be treated as if they were capital expenditures and embodied in what is defined as I_g in the present model.

[15]Also in this paper, in order to focus the attention on fiscal policy, the possibility to monetize the debt is not contemplated. Moreover, like in Domar's model, the interest rate on the debt and prices are taken as given and constant.

determination of the changes in a and I_g that yield a change in $(r-g)b$ equal to $-D$ while $(\tau-\gamma)$ is left unchanged. Formally,

$$\frac{d}{da}\left[t-at-\frac{I_g(\Gamma-at)}{I_g+I_p)}\right]da+\frac{d}{dI_g}\left[t-at-\frac{I_g(\Gamma-at)}{I_g+I_p}\right]dI_g=0$$

(7)

$$\frac{d}{da}\left[r-(\Gamma-at)\sigma\right]\frac{B(\Gamma-at)}{I_g+I_p}da+\frac{d}{dI_g}\left[r-(\Gamma-at)\sigma\right]\frac{B(\Gamma-at)}{I_g+I_p}dI_g=-D$$

That is to say, the total differential of the right-hand side of (6) must be equal to zero, whereas the total differential of the left-hand side has to decrease by an amount D

The general solutions of (7) are:

$$dI_g=\frac{D\left(I_g+I_p\right)^2}{B(\Gamma-at)^2\sigma} \tag{8}$$

and

$$da=-\frac{(\Gamma-at)dI_g}{t\left(I_g+I_p\right)} \tag{9}$$

If the ratio of the primary budget to the GDP has to remain unchanged, da and dI_g must be such that (8) and (9) are fulfilled. If the economy's growth rate has to increase, a must decrease (da must be negative). This is ensured by dI_g being positive. dI_g, is increasing in D and decreasing in B. The larger is the required reduction in the debt ratio (D), the larger is the required increase in g, the larger is the required decrease in the share of current expenditures (a) and, hence, the larger is the required increase in capital expenditures. In contrast, the larger is B, the larger is the impact that the change in the growth rate has on the left-hand side of (6).

However, not every decrease in a is economically acceptable and compatible with the hypotheses of the model. The decrease in a must be such as to ensure that

$$a+da>0 \tag{10}$$

In fact, it has been assumed that there must always be a positive amount of current expenditures.[16] Therefore, the additional constraint (10) has to be introduced. Such a constraint, which essentially is a limit to the variations of a and I_g, can be expressed in different ways. It is interesting to express it in terms of D, that is to say as a limit to the size of the reduction of the growth of the debt ratio. In order that $(a+da>0)$, it must be

$$D<\frac{aBt\left(\Gamma-at\right)\sigma}{I_g+I_p} \tag{11}$$

The rationale of this constraint is simple. From (8), the larger is D the larger is dI_g and, hence, the larger is the reduction in a. But a cannot decrease freely as it must remain positive. The constraint on D, therefore, amounts to a constraint on the reduction of a. At the same time, as dI_g is decreasing in B, the larger is the latter the smaller is the

[16]Moreover, if $(a+da)$ were allowed to be negative, this would amount to allow an increase in the tax rate t, which instead is kept constant throughout.

increase in capital expenditures (the smaller is the decrease in a) and, hence, the larger can D be.

From this a conclusion follows: governments largely indebted can more easily adjust the dynamics of their debt through changes in the composition of their expenditures rather than through running primary surpluses that require a generalized reduction in spending and/or increases in taxation. This, however, is true only if the initial share of total revenue devoted to current spending (a) is sufficiently large. As the constraint on D is increasing in a, the larger is the latter the less stringent is this constraint. Intuitively, if a is initially very small, it might be impossible to reduce it to the extent required to raise the growth rate above the interest rate to obtain $b(r-g) = -D$. Thus, for governments that have already a small share of their revenue devoted to current spending it could be impossible to realize a large reduction of the debt ratio by leaving the ratio of their primary budget to GDP unchanged.

Therefore, there may exist situations in which it is impossible to stabilize a fast increasing debt ratio (a large D) through policies that do not affect the primary budget. In such cases, either the stabilization of the debt ratio is only partial, i.e. the rate of growth of the ratio of the public debt to GDP is reduced but not brought to zero,[17] or the stabilization must be realized by also making the fiscal budget vary. However, as it is shown below, the choice to stabilize the debt ratio also through variations of the ratio of the primary fiscal budget to GDP does not necessarily ensure that the constraint on D becomes less restrictive.

In order to show this, let us consider a case in which the debt stabilization is realized through both an increase in $(\tau - \gamma))$ and a decrease in $(r-g)$. The problem to solve, therefore is

$$\frac{d}{da}\left[t - at - \frac{I_g(\Gamma - at)}{I_g + I_p}\right] da + \frac{d}{dI_g}\left[t - at - \frac{I_g(\Gamma - at)}{I_g + I_p}\right] dI_g = mD$$

$$\frac{d}{da}\left[r - (\Gamma - at)\sigma\right] \frac{B(\Gamma - at)}{I_g + I_p} da + \frac{d}{dI_g}\left[r - (\Gamma - at)\sigma\right] \frac{B(\Gamma - at)}{I_g + I_p} dI_g = -nD \tag{12}$$

with $0 \leq m \leq 1$ and $n = 1 - m$

The general solutions for (12) are:

$$dI_g = \frac{D}{B\sigma} \frac{[nI_p + Bm(r - 2g)]}{I_p} \left(\frac{I_g + I_p}{\Gamma - at}\right)^2 \tag{13}$$

and

$$da = -\frac{dI_g}{t}\left(\frac{\Gamma - at}{I_g + I_p}\right) - \frac{(I_g + I_p)\, Dm}{I_p t} \tag{14}$$

If it is imposed that $a + da > 0$, we need that

$$D < \frac{aBtI_p\sigma}{nI_p + mB[r - (\Gamma - at)\sigma]}\left(\frac{\Gamma - at}{I_g + I_p}\right) \tag{15}$$

[17] D in (7) is reduced, so that (11) is fulfilled.

If the new constraint on D must be less stringent than the one when the debt ratio adjustment is realized by leaving the primary budget ratio unvaried, the ratio of (15)) to (11)) must be larger than 1. It is easily seen that this condition is fulfilled for any

$$B < \frac{I_p}{r - (\Gamma - at)\sigma} \tag{16}$$

Thus, in a situation in which a targeted reduction D of the debt ratio cannot be realized by leaving the primary budget ratio unchanged, it can be possible to achieve the target by implementing a mixed policy, i.e. variations of the fiscal budget ratio and of its composition. However, this alternative policy is feasible only if B is sufficiently small to fulfill (16). In fact, if (16)) is not fulfilled, the constraint on D is less stringent when the primary budget ratio is left unchanged, so that the reduction of the rate of increase of the debt ratio that can be realized is larger if the fiscal budget ratio is left unchanged.[18]

In conclusion, under the hypothesis that the economy's growth rate is dependent on the composition of the public expenditure, it is possible to show that, starting from a situation in which the interest rate is higher than the growth rate, appropriate changes in the composition of the public expenditure, namely a reduction in the share of the fiscal revenue devoted to current spending and an increase in public investment, determine an increase in the growth rate, which becomes higher than the interest rate. As a consequence of this, the stabilization of the ratio of the public debt to the GDP does not require the running of a primary fiscal surplus. When $r < g$, there is no need to have $\tau > \gamma$ in order that (3) is fulfilled. The possibility to implement such a policy is greater when the government is largely indebted and, initially, it devotes a large share of its revenue to current spending.

As the debt stabilization is essentially realized by varying the differential between r and g, the same results could be obtained by implementing a monetary policy that lowers the interest rate while the growth rate is left unchanged. The choice to concentrate on changes in the growth rate derives from the present model being inspired by Domar's approach, which underlines the importance of considering the effects of public spending on the economy's growth rate.

5.2. **The average productivity of investment is an increasing function of public investment.** So far, the analysis was carried out under the assumption that the productivity of investment (σ) is constant and it is the same for public and private investment. Now this hypothesis is lifted. It is assumed that $\sigma_g \neq \sigma_p$, where σ_g is the productivity of public investment and σ_p is the productivity of private investment.[19]

Here, in particular, it is hypothesized that the productivity of private investment (σ_p) and, hence, the average productivity σ are an increasing function of public investment. The rationale of such hypothesis can be that a significant part of public investment is

[18]It is interesting to notice that the same condition (16) above applies even if the stabilization is realized only through changes in the fiscal budget ratio, i.e. when $m = 1$ and $n = 0$. In this case too, as it can be easily verified, the constraint on D is less stringent only upon the fulfillment of (16).

[19]If the productivity of public and private investments is different, the average productivity σ is a weighted average of σ_p and σ_g: $\sigma = \frac{\sigma_g I_g + \sigma_p I_p}{I_g + I_p}$. It is obvious that if $\sigma_g > \sigma_p$, σ is increasing in I_g.

directed to infrastructures (e.g. networks, means of communication, etc.) that raise the overall efficiency of private capital expenditures. Under this hypothesis, it is

$$\sigma = f(I_g)$$

with $\frac{d\sigma}{dI_g} > 0$.

We now return to look at the problem of debt stabilization under the new hypothesis on σ. For simplicity, we consider only the case in which the policy objective is to stabilize the debt ratio by leaving the primary fiscal budget unchanged and assume that σ is linearly increasing in I_g, i.e. that it is

$$\sigma = kI_g \tag{17}$$

with $k > 0$.

The problem to solve now is

$$\frac{d}{da}\left[t - at - \frac{I_g(\Gamma - at)}{I_g + I_p}\right] da + \frac{d}{dI_g}\left[t - at - \frac{I_g(\Gamma - at)}{I_g + I_p}\right] dI_g = 0$$

$$\frac{d}{da}\left[r - (\Gamma - at)kI_g\right] \frac{B(\Gamma - at)}{I_g + I_p} da + \frac{d}{dI_g}\left[r - (\Gamma - at)kI_g\right] \frac{B(\Gamma - at)}{I_g + I_p} dI_g = -D \tag{18}$$

The solutions for dI_g and da are:

$$dI_g = \frac{DI_p}{BkI_p(2I_g + I_p)} \left(\frac{I_g + I_p}{\Gamma - at}\right)^2 \tag{19}$$

and

$$da = -\frac{dI_g}{t}\left(\frac{\Gamma - at}{I_g + I_p}\right) \tag{20}$$

In order that $a + da$ is positive, a constraint on D must be added:

$$D < \left(\frac{\Gamma - at}{I_g + I_p}\right) aB(2I_g + I_p)kt \tag{21}$$

The change of the hypothesis about σ does not affect the right-hand side of (6)), only its left-hand side. More precisely, what changes is the effect of a change in I_g. Now I_g has to increase less (a has to decrease less) to realize the stabilization of the debt ratio because it also directly affects the productivity of investment and, hence, the growth rate. It can be easily verified that dI_g in (19) is smaller than dI_g in (8). Finally, it is also clear that the constraint on D is now less stringent.[20] Thus, in conclusion, when σ is increasing in I_g the effect of a change in the composition of public spending in favor of capital expenditures is more powerful.

5.3. **A graphical representation.** A graphical representation of the results obtained above can be helpful. Figure 1 below illustrates the results of the analysis of the case in which the ratio of the primary fiscal budget to GDP is left unvaried and σ is constant and independent of I_g Each point on the line FF in Figure 1(a) represents a combination of the ratio of current spending to total revenue (a) and public capital expenditure (I_g);

[20] As $aB(2I_g + I_p)kt > aBt\sigma$.

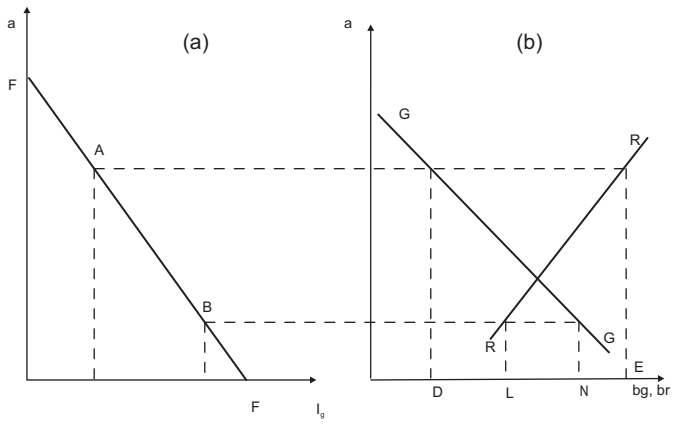

FIGURE 1

all these combinations are associated with the same value of the ratio of the primary fiscal budget to GDP. In other words, each point on the line FF is obtained by varying the initial values of a and I_g (subject to the constraint that $a + da > 0$) in such a way that $(\tau - \gamma)$ remains constant. If, for example, the point A with coordinates I_g and a represents the initial position of the economy, the point B has coordinates $I_g + dI_g$ and $a + da$ where dI_g and da are the solutions of (7) or (18)) above subject to the constraint (10) or (21)). In turn, each point on the line FF is associated to a certain level of income, Y, a certain value of $b = \frac{B}{Y}$ and a certain growth rate g.

As the composition of public spending moves in favor of capital expenditures (e.g. from A to B in Figure 1(a)) Y increases, br decrease and bg increases.[21] The decrease in br is reflected by the line RR in Figure 1(b) being upward sloping; the increase in bg is reflected by the line GG in Figure 1(b) being downward sloping.

Let us now examine the effects of changes in a and I_g by considering a case in which, in the initial situation (point A), there is a primary fiscal deficit and the ratio of the public debt to Y (b) is rising because it is $b(r - g) = \text{DE} > 0$. If the composition of public spending changes from A to B, the ratio of the fiscal deficit to Y does not change, but now it is $b(r - g) =$ - LN. If - LN is equal to $(\tau - \gamma)$ the debt ratio is stabilized. If, in B, it is still $\tau - \gamma) <$ LN, the debt ratio can be stabilized by moving further down along the line FF, i.e. by further increasing the modulus of $b(r - g)$.

6. Conclusion

The main object of this paper is to present an approach to the problem of the stabilization of the ratio of the public debt to GDP that differs from the mainstream approach. The ratio of the public debt to GDP can be analyzed by considering the growth rate of the economy as independent of the composition of public spending. A smaller share of public spending devoted to 'unproductive expenditures' positively affects the growth rate and,

[21] In general, as Y is increasing in I_g and in a, an increase in I_g associated with a decrease in a could produce a negative effect on Y. However, if dI_g and da are the solutions of (7) or (18) above, it can be easily verified that the effect on Y is positive. The fact that the variations in the public investment and in the public propensity to consume are the solutions of (7) or (18) also ensures that bg increases.

hence, the ratio of public debt to GDP. The present model is built in such a way to emphasize this aspect. Other functional relations–like, e.g., the tax rate–are dealt with in a very simple way, but the model could be further developed. The model can also be extended by considering an open economy and the possibility that part of debt stock is owned by the foreign sector.

The present work draws its inspiration from two contributions by Domar in the 1940s. The analytical relationship between the composition of public spending and the economy's growth rate is derived from a generalization of Domar's growth model. The idea that the problem of the public debt must be dealt with by considering the effects on the GDP of different types of public expenditure comes from Domar's contribution on the burden of the public debt. In this sense, the paper offers an 'old answer' to the old question of fiscal sustainability.

Mainstream analyses of fiscal sustainability are concerned about the negative effects of a large and increasing ratio of the public debt to the GDP but they do not seem to be concerned about the problem of the effects of government spending on the economy's output. In fact, in these models, the economy's growth rate is totally independent of the level and composition of the public expenditure as well as the public debt[22] The mainstream approach to fiscal sustainability amounts to not offering any real explanation of why the government borrows resources from private agents, unless it is (at least implicitly) accepted the idea that governments draw resources from the private sector only for the benefit of their bureaucratic apparatuses and/or their constituencies, without any concern for the general welfare. The exclusive concern for the stabilization of the debt ratio through the realization of primary surpluses follows from this sort of approach. Differently, this paper suggests that the problem of debt stabilization should not focus exclusively on the realization of primary surpluses but rather on the effects that different types of public expenditure have on the economy's overall productivity and the growth rate.

The approach to the problem of the public debt suggested here does not imply that there should not be any concern for the state's debt and, in particular, for its possible tendency to explode. However, the concern for a continuously growing ratio of the public debt to GDP should not derive from the fact that it will cause the tax burden to increase continuously or from the unlikely possibility of a default of the state, but rather from the fact that a growing debt ratio indicates that the government is not using resources in an 'efficient' manner.[23] The government is spending resources in such a way that the GDP does not grow sufficiently to keep the debt ratio stable. In this perspective, policies aimed at reducing the debt ratio should be centered on the attempt to increase productive expenditures rather than being concentrated on measures aimed at eliminating deficits through reducing expenditure and ignoring the long-period negative effects of such reductions.[24]

[22]Even the obvious multiplier effects of public spending on the GDP are often ignored, for example by assuming that any reduction in the level of public expenditure implies a proportional reduction in its ratio to the GDP.

[23]In an open economy, there would be the additional concern that a growing debt ratio implies that a growing flow of internal resources must be devoted to the servicing of the external debt.

[24]The EMU and its fiscal discipline are evident examples of this wrong approach to the problem of public debt. In fact, several economists have been suggesting that the EMU adopt some version of the

However, it is necessary to conclude with some qualifications. First of all, it is important to stress that the paper develops the analysis of fiscal policy and public debt in the context of a long-period equilibrium model. The concern for the debt stabilization in the long period does not exclude that, in the short period, the ratio of the public debt to GDP can, or must, rise to stabilize the economy. The short-period implication of the present model, however, is that, if it is necessary to have an increase in the primary deficit, it is preferable to do so by increasing capital rather than current public expenditures, as the former have also a positive impact on the economy's growth.

Secondly, as the present analysis is carried out by using an equilibrium model, the results obtained should be regarded as strictly belonging to the domain of comparative dynamics. Given two economies that differ only for the ratio of public current expenditure to the tax revenue, the economy with the smaller ratio grows at a higher rate and can stabilize the ratio of the public debt to the GDP without necessarily running primary surpluses. The present equilibrium model does not permit any rigorous analysis of dynamic changes in an economy that varies the composition of its public expenditure over time. In the present context, considerations of this nature can be made only at an intuitive, casual level.

Finally, it should also be stressed the need for further developments to eliminate some of the simplifications and assumptions made in the present model. In particular, our model is based on the assumption that a clear-cut distinction between capital and current expenditures can be made as well as on the idea that no current expenditure has positive effects on the economy's productivity. These clearly are simplistic assumptions. Domar himself underlines that productive public expenditures are not necessarily to be identified with capital expenditures. Some classes of current expenditures have a positive impact on the economy's productivity while some sorts of unproductive expenditures can be too easily disguised as capital expenditures. Further developments and, above all, attempts to look at the problems of fiscal and debt sustainability in actual situations should be based on further refinements concerning the classification of the government expenditures and their impact on the economy as a whole. More in particular, a more refined analysis can be carried out by introducing the notion of human capital and by considering the public current expenditures that contribute to the formation of it (for example, the expenditures on education) as investment.

UK "golden rule", which deals with government deficits by distinguishing between current and capital expenditures.

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